Laboratory 5
Measurement of DC and AC voltages
rev. 9.2e

Purpose: Getting acquainted with measuring AC and DC voltages, and the differences between particular values.

Summary of theory

Parameters of periodical AC signals

Let there be a periodic function (signal) with the period T, $x(t) = x(t + kT)$. The following quantities can be defined for this signal:

- **Peak value** – the extreme value (positive or negative) of the signal (U_{pk+}, U_{pk-}).
- **Peak-to-peak value** – the change between the positive and negative peak values of the signal.

$$U_{pk-to-pk} = U_{pk+} - U_{pk-}$$

- **Mean value** – the DC component of the signal, or the average value:

$$\overline{u(t)} = U_0 = \frac{1}{T}\int_t^{t+T} u(t)dt$$

is the value indicated by a magneto-electric instrument, if the frequency f is much higher than the frequency that the needle (pointer) of the instrument can "follow". It is not very useful since most of the common signals are symmetrical and, thus, their mean value is 0.

- **Average rectified value** or **Absolute mean value** – the average value of the rectified signal. It can be defined for *half-wave rectified* signals (the negative portions of the waveform are blocked) and for *full-wave rectified signals* (the negative portions are converted into positive ones). It is denoted U_{ma} or, sometimes U_m, because, as it was said before, the mean value is rarely used.
  - For *full-wave rectification:*

$$U_m = \overline{|u(t)|} = \frac{1}{T}\int_t^{t+T} |u(t)|dt$$

  - For *half-wave rectification*- positive polarity

$$u_+(t) = \frac{1}{2}\left(u(t) + |u(t)|\right) \Rightarrow U_{m+} = \overline{u_+(t)}$$

  - For *half-wave rectification*- negative polarity

$$u_-(t) = \frac{1}{2}\left(u(t) - |u(t)|\right) \Rightarrow U_{m-} = \overline{u_-(t)}$$

- **Root Mean Square (RMS) value** – The RMS voltage of an *AC* signal is the value of the *DC* voltage that yields the same power dissipation as the *AC* voltage.

$$U_{RMS} = \sqrt{\frac{1}{T}\int_{t_i}^{t_i+T} x^2(t)dt} = \sqrt{\overline{x^2(t)}}$$

Peak, mean, average rectified (*full wave rectification* FWR and *half-wave rectification* HWR) and RMS values, for common periodic signals with the amplitude A, are given in Table 1.

waveform	U_{pk}	$\overline{u(t)}$	U_m HWR	U_m FWR	U_{RMS}
sine	A	0	A/π	2A/π	$A/\sqrt{2}$
rectangular (symmetrical)	A	0	A/2	A	A
triangular (symmetrical)	A	0	A/4	A/2	$A/\sqrt{3}$

Table 1

The following factors are defined:

- *the form factor (FF or K_f):* $K_f = \frac{U_{RMS}}{U_{ma}}$
- *the crest factor (CF):* $CF = \frac{U_{pk}}{U_{RMS}}$

Their values for common (symmetrical) waveforms can be calculated from table 1 and are given in table 2.

Waveform	K_f		*CF*
	full-wave rectification	half-wave rectification	
sine	1.11	2.22	1.41
rectangular (symmetrical)	1	2	1
triangular (symmetrical)	$\sqrt{3}/4$	$\sqrt{3}/2$	$\sqrt{3}$

Table 2

The table allows for deducing a value based on any other value, but it can be seen that this cannot be used unless the waveform is known!

Devices used for voltage measurements:

- ***DC voltmeter:*** measures a DC signal's voltage or the mean value of an alternating signal applied at its input:

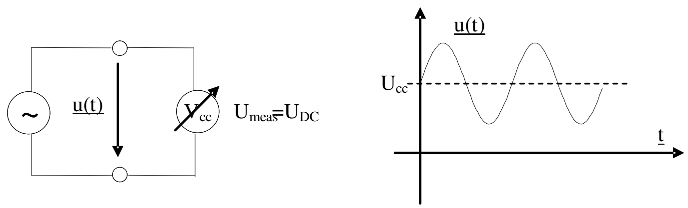

Figure 1: DC Voltmeter

- ***AC voltmeter*:** measures *one* of the particular values associated with an alternating signal; most commonly, when not otherwise specified, the RMS value is measured, because this has the highest practical utility (e.g., the AC Line voltage has a peak value of 311V, RMS value of 220V, or peak-to-peak value of 622V; which one of these values sounds more familiar?)

Constructively, there can be two types of AC voltmeters:

1*) True RMS Voltmeters*, usually marked with inscriptions such as *True RMS*, *RMS Responding,* etc. These types of voltmeters measure the RMS value of the signal, regardless of its waveform, usually by means of calculations performed internally, or using the thermal effect. They are somewhat more expensive.

2) *Average Responding Voltmeters, graded in RMS values for a sine wave*, but which do not measure the RMS value of the signal **directly**. This method is used because the average rectified value of a signal is easier to obtain (with a simple diode or diode bridge for high-value signals) than the RMS value, and so the device is less expensive. So, Average Responding Voltmeters measure the average rectified value of the signal which is then converted into the RMS value for a **sine wave**, by using the form factor for a sine wave from table 2:

$$k_F^s = \frac{U_{ef}^s}{U_{ma}^s} \Rightarrow U_{ef}^s = k_F^s \cdot U_{ma} = 1{,}11 \cdot U_{ma}$$

Therefore, the device measures the average rectified value of the input signal:

$$U_{mas} = U_{ma}$$

and indicates a value that is 1.11 times higher:

$$U_{ind} = k_F^s \cdot U_{ma} = 1{,}11 \cdot U_{mas}$$

Disadvantage: it can be noticed that the device measures the RMS value correctly *only* for a sine wave, the only type of signal for which $K_f = 1.11$. ***The device commits a systematic error for any other types of signals.***

One wishes, in certain situations, to express the RMS voltage in terms of a comparison with a reference voltage. E.g.:

- $U_{ref\,1}$= 1V :

$$|U|dB = 20 \cdot \lg \frac{U}{U_{ref\,1}}$$

This voltage corresponds to a reference power P_{ref} = 1mW dissipated on a resistance of 1kΩ (1000Ω). Therefore 0dB = 1V.

- $U_{ref\,2}$= 0.775V :

$$|U|dBm = 20 \cdot \lg \frac{U}{U_{ref\,2}}$$

This voltage corresponds to a reference power P_{ref}= 1mW dissipated on a resistance of 600Ω (this value is the standard impedance of telephone lines). We can calculate 0dBm = 0.775V.

Measurements

1. ***Measuring the RMS voltage for a sine wave.*** Generate a sine wave of amplitude *U* (written on the blackboard) and frequency 1kHz, by using the signal generator. The amplitude is adjusted from the generator. Use **CH1** on the oscilloscope and choose a vertical deflection coefficient, C_y which maximizes the vertical size of the waveform. Use a horizontal deflection coefficient of C_x=500µs/div. How many divisions (N_y) does the signal amplitude occupy on the scope?

- Measure the signal by using the analog AC millivoltmeter (U_{va}). What is the value indicated by the device (in volts)?
- Measure the RMS voltage of the signal by using the digital multimeter (U_{md}), and selecting the proper type of voltage to be measured – DC or AC - press the **ACV** key.
- Use the ***Measure*** menu from the oscilloscope to measure the RMS voltage ($U_{c.rms}$) and the peak value ($U_{pk\text{-}pk}$) of the signal. Use the following menu entries: **MEASURE → SOURCE=CH1 → TYPE=Cyc RMS** and **TYPE=Pk-Pk** respectively. What is the relationship between the amplitude of the signal and its peak value?
- Compute the theoretical **RMS** voltage for the given signal ($U_{rms.calc}$) based on the value of the signal amplitude *U*. What formula do you use?
- Compute the errors committed by *each* of the three RMS measurements with respect to the theoretical RMS value that you computed ($U_{rms.calc}$):

$$\varepsilon = (|U_{RMS\ measured} - U_{RMS\ calc}| / U_{RMS.calc}) \cdot 100\%$$

2. ***Measuring the RMS voltage for rectangular and triangular waveforms.*** Perform the same measurement and calculations from section 1 but this time for a symmetrical triangular waveform of amplitude U_{tr} and a rectangular waveform of amplitude U_{dr} (duty cycle η=50%), with frequencies of 1kHz. What calculation formulas do you use for the RMS value as a function of

the amplitude? Notice which of the instruments commit higher errors than those for a sine wave. Explain why.

3. ***Measuring the RMS voltage level* (*in dB*).** Generate a sine wave with an amplitude of U and a frequency of 1kHz , by using the signal generator. Use a vertical deflection coefficient of the value with which the signal occupies as much of the screen as possible and adjust the amplitude from the generator until the requested value is reached.

- Measure the voltage level of the signal both in dB and dBm, by using the AC millivoltmeter (U_{va}[dB], and U_{va}[dBm]) and the digital multimeter (U_{md}[dB] and U_{md}[dBm]).

***Remark*:** *For the digital multimeter, the secondary display in dBm can be activated by pressing* **SHIFT dBm** *while in* **ACV** *mode; for measuring levels in decibels* (dB) *a reference resistance of* 1000 Ω *has to be selected*, *for measuring in* dBm *select a reference resistance of* 600 Ω. *The values can be modified by pressing:*
SHIFT SET →Ω →*value of* 600Ω , 1000Ω (*use arrow keys* ▲▼) → **SET**

Pay attention! Regardless of the selected value, the same indication lights up beneath the display „dBm". Therefore, the only way of knowing what voltage level the instrument indicates is by checking the value of the reference resistance. Notice the advantage of the analog instrument, for which a scale can be easily drawn for each desired indication!

- Compute in V the theoretical value of the RMS voltage level (based on the values of the specified amplitude) then transform it in dB, dBm (U_{calc}[dB] , U_{calc}[dBm])

Remark: take into account the reference resistance for each type of voltage level.

Explain why the voltage level, expressed in dB/dBm, changes even though the RMS value remains the same.

4. ***Measuring the DC level of a sine wave.***

a) Generate a sine wave with an amplitude of U_o=1,5V, a DC component of U_{DC}=+1V and a period of $T=250$ µs. Check that the DC component is correctly set by viewing by how many vertical divisions the waveform goes up or down when the display of the oscilloscope is changed from **CH1 Menu**->**Coupling->AC** (display without DC) to **DC**. The settings for the oscilloscope will be:
- C_y=0,5V/div and C_x=50 µs/div,
- **Coupling->DC**,
- adjust the 0V level, from **VERTICAL POSITION** (the GND level – indicated on the left side by the arrow with the channel number next to it) to 2 divisions above the lower end of the scope. View and draw the waveform.

Compute the theoretical U_{V+} and U_{V-} (notice how, when $U_{DC} \neq 0$, $U_{V+} \neq U_{V-}$). Use the **MEASURE** menu from the oscilloscope, and measure U_{V-} , U_{V+} and the mean value (**MEASURE → SOURCE=CH1 → TYPE= Min, Max** or **Mean**). Measure the DC component of the signal by using the digital multimeter (press the key **DCV**) and compare it to the one measured with the oscilloscope.

At the end of the measurement, reposition the GND level at the middle of the scope screen.

b) We will now illustrate a situation when the MEAN value indicated in the MEASURE menu does not correspond with the theoretical one. This happens when the scope does not display an integer number of periods. It is important to understand that the oscilloscope computes the MEAN only for the **displayed image**, while, normally, when we talk about the mean value of a signal, we refer to the **average value** of the signal **over one period**!

Move from **HORIZONTAL POSITION the arrow that corresponds to the beginning of the signal display,** at the left edge of the screen. Set TRIGGER LEVEL=0V. Apply a ***sine wave*** with f=1KHz, A=2V, using C_X=250us/div. Use the MEASURE menu to determine the MEAN value. How many periods are there on the scope? Draw the image.

Modify the display to the „invert" mode. Go to **CH1 MENU → Invert → On.** Use the MEASURE menu to determine the new MEAN value.

Explain why the MEAN value changed, although the signal was not altered (nobody operated the generator!)

Modify *Cx* to 100µs/div. Determine the new MEAN value. How many periods are there on the scope this time?

Explain the obtained results. Which of the MEAN values correspond to the mean value of the signal and why?

Return the display to **CH1 MENU → Invert → Off.**

Remark: In the measure menu, the RMS value is abbreviated **Cyc RMS** because, in the case of this value, the scope performs the computation for one period of the signal (*cycle*), no matter how many periods are displayed on the screen.

5. ***Measuring a half-wave rectified sine wave.***

a) Build the circuit from figure 2 on the solderless breadboard. The resistance is not part of the rectifier, it is merely a *load resistance* (this way, the circuit does not function without a load, which would be something unseen in common practice and may distort the signal, because too little current passes thru the diode).

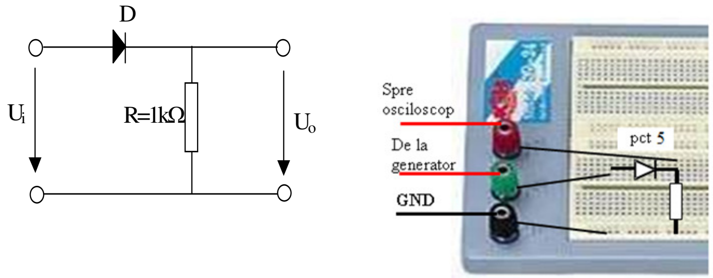

Figure 2: The schematic of the half-wave rectifier; realization example
OBS: for the diode, the vertical line on the symbol correspond to the strip that is marked on the real component

- Generate a sine wave with an amplitude of 5V and a frequency of 1kHz, without a DC component, and apply it at the input of the rectifier. Make sure that you have **CH1/2 Menu -> Coupling->DC** for both channels.
- Simultaneously view the rectifier input signal on CH1 and the output signal on CH2 by using the oscilloscope with both vertical deflection coefficients C_y=5V/div. Draw the two signals on the same chart. Measure the DC component of the output signal (U_o) using the DC voltmeter mode of the digital multimeter (press key **DCV**) ($U_{DC\ HWR}$).

Indication: The real diode introduces a voltage drop of approximately 0.6V, which makes the signal obtained on the screen differ a little from the ideal half wave rectified signal.

- Compute the theoretical value of the DC component of the signal ($U_{th\ DC\ HWR}$).

b) Repeat point a), but reverse the position of the diode, so that the negative polarities are rectified (negative half-wave). Draw again, on the same chart, the two signals (input and output, in different colors).

6. *Measurements for the series/parallel peak detector*

A *peak detector* allows measuring the peak value of the voltage that is applied at the input.

a) Apply at the input of the circuit from figure 3a a sinusoidal signal of amplitude (peak value) 5V and frequency 10kHz, with null DC component.

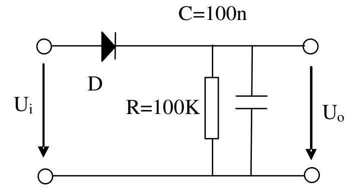

Fig. 3a: series peak detector

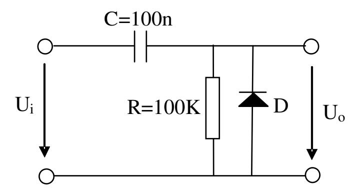

Fig. 3b: parallel peak detector

For each circuit:
- View simultaneously on the 2 channels of the oscilloscope (both Cy = 5V/div) the voltages Ui and Uo from the input and output of the circuit.
- Verify that the 2 zero levels are in the middle of the screen. In this way, we can easily compare the DC components of the 2 signals. Cx will be set so that on the screen we see 2-3 periods.
- Connect at the output (Uo) the digital multimeter, set on mode DC voltmeter (**DCV** key). What value does the voltmeter measure? Compare with the peak value of the input voltage. What do you observe? Why is the circuit called peak detector?

Indication: take into account the voltage drop on the diode.

b) verify the functioning of the series peak detector in the presence of a DC component: add a DC component of 2V over the AC signal of amplitude 5V (unmodified). What value does the DC voltmeter indicate? How does the image modify?

c) Repeat the observations and measurements from a) on the circuit in fig. 3b (parallel peak detector). Use the same signal as for a), of amplitude 5V, null DC component.

Based on the image at the output, why do you think that the parallel circuit is called peak detector and also "voltage-shifter"?

Indication: the parallel circuit produces at the output a variable signal (the shifted input signal), while the series circuit produces a continuous signal; obviously, the 2 signals are very different, but their measurement using a DC voltmeter, which computes the *mean* of the measured signal, allows to obtain the observed effect and justifies the fact that both circuits are called *peak detector.*

d) Repeat point b) on the parallel circuit, adding a DC component of 2V.

Based on the different way in which they behave in the presence of a DC component added to the AC signal (points b, d), which of the detectors (series or parallel) do you think is preferable and why?

Preparatory questions

1. Consider the sine wave $s(t)=5\sin(\omega_0 t)[V]$. Compute the RMS voltage, the peak voltage and the average rectified voltage.

2. Consider a symmetrical rectangular signal with an amplitude of A=2V. Compute the RMS voltage, the mean voltage and the average rectified voltage.

3. Consider a symmetrical triangular signal with an amplitude of A=3V. Compute the RMS voltage, the mean voltage and the average rectified voltage.

4. Explain the difference between a True RMS AC voltmeter and an Average Responding voltmeter graded in RMS values for a sine wave.

5. The signal $s(t)=3+2\sin(\omega_0 t)+2\sin(3\omega_0 t)\ [V]$ is measured with a DC voltmeter. Determine the indication of the instrument..

6. The voltage $s(t)=4+3\sqrt{2}\sin(\omega_0 t)\ [V]$ is measured with an AC voltmeter. Determine the indication of the instrument.

7. The voltage $s(t)=2\sqrt{2}\sin(3\omega_0 t)[V]$ is measured with an AC voltmeter. Determine the indication of the instrument.

8. Given the voltage U=7,75V, compute the voltage level expressed in dBm.

9. Given the voltage U=20V, compute the voltage level expressed in dB. *Indication* $\lg 2 \approx 0{,}3$.

10. A voltage has the value of U=32dB. Determine its value expressed in volts. *Indication* $\lg 2 \approx 0{,}3$.

11. A voltage has the value of U=60dBm. Determine its value expressed in volts.

12. Compute the mean value of the signal $s(t)=2\sin^2(\omega_0 t)[V]$.

13. Given the voltage of U=4,48V, compute its value, expressed in dB. *Indication* $\lg 2 \approx 0{,}3$.

14. A voltage has the value of U=26dBm. Determine its value expressed in volts. *Indication* $\lg 2 \approx 0{,}3$.

15. Which component(s) must be changed in order to transform a series peak detector into a parallel one?

Exercises :

1. Compute the average rectified voltage and the RMS voltage of the following signals (write the computations in detail):

$$s_1(t)=\left|3\sin\left(10^4\pi t\right)\right|$$
$$s_2(t)=2\sin(2000\pi t)$$
$$s_3(t)=3+\cos(2000\pi t)$$
$$s_4(t)=2(\sin 4000\pi t)^2$$
$$s_5(t)=\sin\left(10^4\pi t\right)-\left|\sin\left(10^4\pi t\right)\right|$$
$$s_6(t)=2\cdot\cos\left(3\cdot 10^4\pi t\right)+2\cdot\left|\cos\left(\cdot 10^4\pi t\right)\right|$$

2. Compute the average rectified voltage and the RMS voltage of the signals in figure 4 (write the computations in detail):

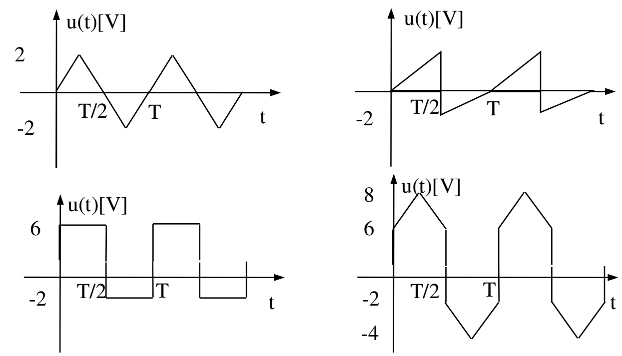

Figure 4: Time-domain representation of signals

3. The voltage represented in figure 5 in measured with a voltmeter that can measure DC voltages and AC voltages, with a full wave rectifier :

- The DC voltage indication is U_1=2V;
- The AC voltage indication is U_2=5,55V.

a) Knowing that the AC voltmeter is graded in RMS values for a sine wave compute voltages E_1 and E_2 if τ=T/2.

b) What will the voltmeter indicate in the two cases if τ=T/3?

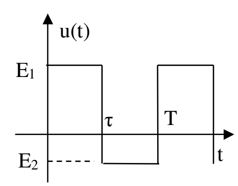

Figure 5: Asymmetrical rectangular waveform