Laboratory 4
Measurements of frequency response

Purpose: Measuring the cut-off frequency of a filter. The representation of frequency responses of some circuits (two-port networks).

Summary of theory

Response of a linear circuit to sine or cosine waves

When a cosine wave is applied at the input of a passive two-port network

$$x(t) = U_i \cos(\omega t + \varphi_x)$$

its response is also a cosine wave, with the expression

$$y(t) = U_i H(\omega) \cos(\omega t + \varphi_x)$$

$H(\omega)$ represents the value of the transfer function (frequency response) at frequency *f*. Taking into account that $H(\omega)$ is a complex quantity, with magnitude $|H(\omega)|$ and argument $arg\{H(\omega)\}$, the real signal $y(t)$ can also be written as

$$y(t) = U_i |H(\omega)| \cos(\omega t + \varphi_y)$$

Observe that the amplitude of the output signal is

$$U_o = U_i |H(\omega)|$$

OBS: The amplitude of the output signal is the PRODUCT between the amplitude of the input signal and the transfer function magnitude.

and the output signal is out of phase with the input signal by the value

$$\varphi_y = \varphi_x + arg\{H(\omega)\}$$

OBS: The phase of the output signal is the SUM between the phase of the input signal and the phase (argument) of the transfer function.

Magnitude of the frequency response $|H(\omega)|$

$H(\omega)$ is measured by applying a sine wave with known frequency *f* and amplitude U_i at the input of the circuit. The amplitude of the output signal, U_0, is measured, and the magnitude of the transfer function at frequency *f* is determined:

$$|H(\omega)| = \frac{U_0}{U_i}$$

If $|H(\omega)|>1$ it is said that the circuit amplifies the signal, if $|H(\omega)|<1$ the circuit attenuates the signal.

$$amplification = |H(\omega)|, \qquad attenuation = \frac{1}{|H(\omega)|}$$

The magnitude of the transfer function expressed in decibels (dB) is denoted as $|H(\omega)|_{dB}$:

$$|H(\omega)|_{dB} = 20lg|H(\omega)| = 20lg\frac{U_0}{U_1} \tag{1}$$

If the voltmeter is calibrated in dB referenced to a voltage U_{ref} (for example U_{ref} = 1V, this yields:

$$|H(\omega)|_{dB} = 20lg\frac{U_0}{U_{ref}} - 20lg\frac{U_i}{U_{ref}}$$

where $20lg\frac{U_0}{U_{ref}}$ is precisely the indication of the voltmeter in decibels (denoted as U_o [*dB*]). Thus

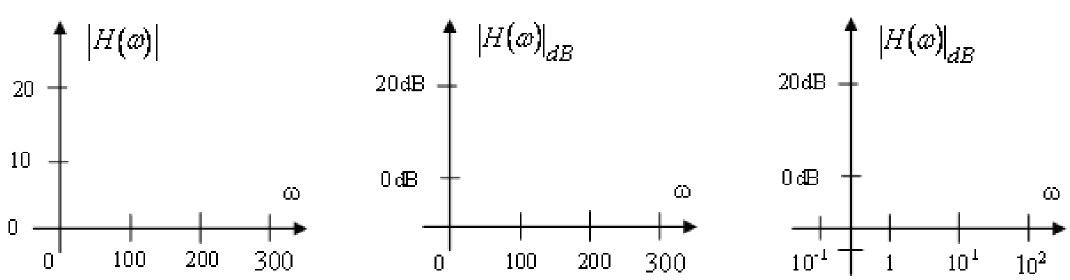
$$\boxed{|H(\omega)|_{dB} = U_0[dB] - U_i[dB]}$$

This is called the frequency response of the circuit and represents the variation of the magnitude of the transfer function with frequency *f*, or angular frequency $\omega = 2\pi f$.

A graphic representation of the magnitude of the frequency response may be in a linear coordinate system, a semi-log one or a double-log one (Figure 1). The third system is preferred. The double-log system, also known as a Bode plot, allows the representation of magnitudes of transfer functions in a large frequency range.

Definitions: The angular frequency range between an arbitrary value ω_1 and $10\omega_1$ is called a ***decade***, and the range between ω_1 and $2\omega_1$, an ***octave***.

Remark: The term ***octave*** may seem confusing since the frequency doubles. It comes from music theory, where an octave means the 8 musical notes comprised in the interval between an arbitrary note and the same note at a frequency 2 times higher. For example, the A note from the first octave = 440Hz, the A from the second octave = 880Hz).



a. linear **b. semilogarithmic** **c. double logarithmic**

Figure 1

For the magnitude of the transfer function, the frequency f_{-3dB} is of high importance. At this frequency the magnitude of the transfer function is lower than its maximum value (in dB) by 3 dB.

$$|H(\omega_{-3dB})|_{dB} = \max\{|H(\omega)|_{dB}\} - 3 \text{ or}$$

$$|H(\omega_{-3dB})| = \frac{\max\{|H(\omega)|\}}{\sqrt{2}} \cong 0.707 \cdot \max\{|H(\omega)|\}$$

Why -3 dB? By using relation (1) one can compute that a decrease of the value of the magnitude by exactly $\sqrt{2}$ times is equivalent to a (approx.) 3dB fall from the magnitude of the transfer function in dB. Likewise, this decrease by $\sqrt{2}$ is equivalent to decreasing the transmitted power by half

To this end, when starting measurements, set the level of the generator so that the voltmeter connected to its output indicates, at the chosen frequency f_0, a 0dB level. Also, use a second voltmeter to monitor the input voltage level. This way, the voltage level from the generator can be fine-tuned in order to maintain the input voltage level at the same value. Choose the frequency f_0 so that the output voltage level does not vary too much with the frequency in the respective range.

Circuits that are studied in this laboratory are given in Figure 2:

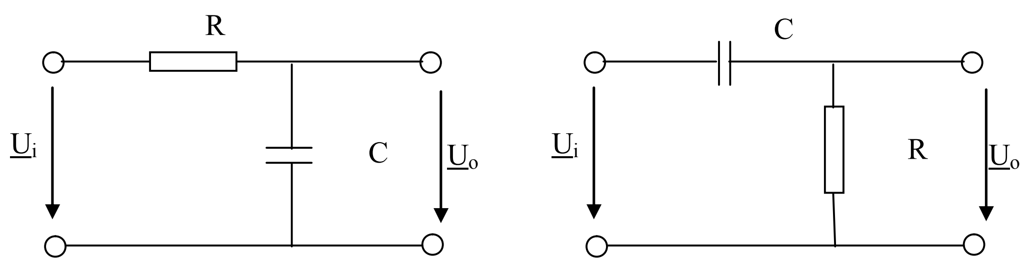


Fig.2: a) LPF –integrator circuit b) HPF - differentiator circuit

The formulae for computing transfer functions of the circuits are:

- Integrator circuit, also named *Low-pass filter (LPF)*

$$H(\omega) = \frac{\underline{U}_o}{\underline{U}_i} = \frac{Z_C}{R + Z_C} = \frac{1}{1 + j\omega RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}, \quad \arg\{H(\omega)\} = -arctg(\omega RC)$$

- Differentiator circuit, also named *High-pass filter (HPF)*

$$H(\omega) = \frac{\underline{U}_o}{\underline{U}_i} = \frac{R}{R + Z_C} = \frac{j\omega RC}{1 + j\omega RC}$$

$$|H(\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}, \ \arg\{H(\omega)\} = \frac{\pi}{2} - arctg(\omega RC)$$

For both circuits, the cutoff frequency (the frequency at which the magnitude of the transfer function falls by 3dB) is given by relation

$$f_t = \frac{1}{2\pi RC} \tag{2}$$

where R and C are the values of the resistor, and the capacitor, from the respective circuit.

Measurements

Remark: Verify that by using the digital multimeter you measure voltages in dB, with a reference resistance of 1000Ω:

- press **SHIFT+SET** (knobs 10 and 11, ANNEX 4).
- press **Ω** (knob 3, ANNEX 4) and set 1000. Press the **SET** knob, again.

In this manner, the level is not expressed in dBm, but in dB (0 dB = 1 V).

(If you would want voltage indication in dBm, you should set 600, using the same method)

Pay attention ! Regardless of the value that you set for the resistance, on the display the indication remains "dBm" (this is a pre-written marking on the display of the device; it doesn't change). Thus, you can know the reference voltage only by using the above method. On an analog millivoltmeter, scales are marked explicitly in dB, dBm respectively.

If the display in dB/dBm is not lit, press **ACV**, followed by **SHIFT+dBm.**

1. Measurement of cutoff frequency for the integrator circuit (low pass filter)

a) Compute the cutoff frequency for the low pass filter circuit from Figure 2a), using formula (2) and the values of the available components **R** and **C**. You can read their values in color code or measure them by using the digital multimeter, in ohmmeter mode and capacitance meter mode.

b) Build the circuit (low pass filter) on the solderless breadboard (see Lab 3 for how the holes of the test board are interconnected). Figure 3 shows the way you should connect measurement devices and the generator. Block D on the figure is the measured circuit. Measure the input signal using the digital multimeter. Measure the output signal with the analog AC millivoltmeter.

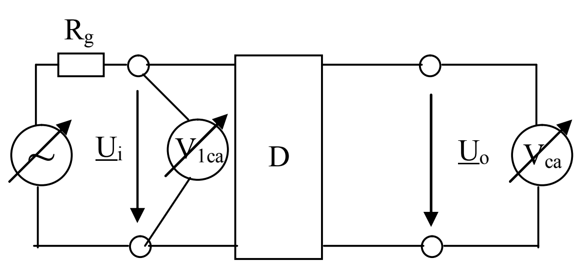


Figure 3: Measurement assembly used for amplitude characteristic.

The integrator circuit is a low pass filter. At low frequencies, the transfer function (the frequency response) is approximately equal to 1 ($U_o = U_i$).

Apply (from the function generator) a low frequency (f_1=1/10 $f_{-3dB\ calc}$) sine wave, without a DC component, at the input of the circuit. Adjust its level (amplitude) from the **AMPL** knob until the digital multimeter indicates $U_i[dB]$ = 0 dB.

Remark: Set the selector switch (3) of the analog AC millivoltmeter on the 0 dB scale ! There is a white stripe on the switch (3). Rotate it precisely to the 0dB indication on the gray area, *not between* two indications.

Measure the level of the output signal, $U_o[dB]$, on the dB scale of the analog AC millivoltmeter. You should find $U_o[dB] = U_i[dB]$ = 0dB.

Increase the frequency of the input signal, using the rotating knob on the generator, and, if necessary, the 2 arrows bellow it, to select the digit that you want to modify. This way, you obtain a coarse or fine adjustment of the frequency) until $U_o[dB]$= –3dB. The obtained frequency is the cutoff frequency of the circuit, f_{-3dB}.

Observe that, while adjusting the frequency, the input signal amplitude can *slightly* change. In this case, fine tune the signal's amplitude from the generator in order to obtain 0dB on the digital multimeter, and adjust again the frequency to have $U_o[dB]$ = –3 dB on the AC millivoltmeter.

Important remark: The effect of the filter is, obviously, to modify the amplitude of the *output* signal, not that of the *input* signal. The amplitude of the input signal is given by the generator. The adjustment of the frequency from the generator should not have any effect on the amplitude of the input signal. This is independently adjusted from the AMPL knob. However, the *input* signal amplitude could have its value modified by a parasitic phenomenon. This can be caused by the variation of the value of the reactance of the capacitor with the frequency, and, thus, the input impedance of the filter varies with the frequency. This impedance comes in parallel with the output resistance of the generator, which has a *nonzero value*. If the generator were an ideal source, with $R_g=0$, this unwanted phenomenon would not appear. In the previous laboratories, the generator was connected only to the oscilloscope, which had a very high impedance (1MΩ). So, this effect did not appear.

If the chosen R-C values are high enough, so that the load of the generator is low, the variation of the input amplitude with the frequency is negligible. In this case, you may not need the voltmeter at the input.

c) Verify the linearity of the circuit with the level of the signal. We know that an equation is linear if the output value is equal to the input value multiplied by a constant which *does not change* regardless of the input value. In case we read the value in dB, the product becomes a sum, so we have to follow if the difference between output-input is the same, no matter the input signal value.

Modify the level (the amplitude) of the input signal, using the **AMPL** knob, at –5 dB, 0dB and +5 dB. In each case, measure the level of the output signal in dB. The input signal should be a sine wave with the frequency of f_{-3dB}, measured at 1b). Modify, when needed, the position of the selector switch of the millivoltmeter - see annex 4. Observe that, when reading in dB, you have to add the value of the indication to the value of the scale. For example, if the switch is on -10dB and the pointer indicates 2dB, the measured value is -8dB.

Explain the result: Is the circuit linear?

2. *Measurement of the amplitude-frequency characteristic for LPF*

a) Determine the magnitude of the transfer function of the integrator circuit (low-pass filter) (Figure 2a). Apply a sine wave with different frequencies f_i given in the table, where f_{-3dB} is the frequency measured at 1b), and the level (amplitude) set at $U_i[dB]$ = 0dB at the input of the circuit. Measure the level of the output signal on the dB scale of the AC millivoltmeter, $U_0[dB]$. The magnitude of the transfer function will be $\left|H(\omega)\right|_{dB} = U_0\big|_{dB} - U_i\big|_{dB}$.

b) Determine the *roll-off* of the filter in the stopband (frequencies *higher* than f_{-3dB}), from the measurements at 2a). Calculate the roll-off of the filter in dB/decade and also in dB/octave. This is done by determining with how many decibels the amplitude of the signal decreases when its frequency becomes 10 times higher, respectively 2 times higher.

c) Plot the variation of the magnitude of the transfer function with the frequency for the integrator circuit on the logarithmic graph. Why is this circuit named a low-pass filter?

3. *Measurement of the cutoff frequency of the differentiator circuit (High-Pass Filter)*

Compute the cutoff frequency of the High Pass Filter (Figure 2b). Build the circuit (High Pass Filter) on the breadboard. As you did at 1b), determine the cutoff frequency. The difference is that you should not start from a low frequency, but from a high frequency such as $f_2 = 10 f_{-3dB\ calc}$. Set $U_i[dB]$ = 0dB and verify $U_o[dB]$ = 0dB. *Decrease* the frequency until you can read $U_o[dB]$ = –3 dB.

4. *Measurement of amplitude-frequency characteristic for HPF.*

a) As you did at 2a), determine the magnitude of the transfer function for the High Pass Filter (Figure 2b). Measure at the frequencies written in the table, where f_{-3dB} is the frequency measured at 3.

b) Determine the roll-off of the filter in the stopband (frequencies *lower* than $f_{-3\mathrm{dB}}$), from measurements at 4a). Calculate the roll-off of the filter in dB/decade and also in dB/octave (determine with how many decibels the amplitude of the signal decreases when its frequency becomes 10 times lower, respectively 2 times lower). Choose any two frequency that satisfy the desired ratio, *in the stopband*.

c) Plot the variation of the magnitude of the transfer function with the frequency for the differentiator circuit. Why is this circuit named a high-pass filter?

5. ***Determining the time response of the Low Pass Filter and the High Pass Filter***

We are going to study why the LPF/HPF circuits are called integrator/differentiator. Reminder from *Fundamentals of electrical engineering*: the time constant of an RC circuit is $\boldsymbol{\tau = R\,C}$.

Apply, sequentially, at the input of the filter, a ***rectangular*** wave with an amplitude of 5V and frequencies of $\{f_1, f_2, f_3\}$ which correspond to periods $\{T_1 = \mathbf{10\,\tau}, T_2 = \boldsymbol{\tau}, T_3 = \boldsymbol{\tau}/\mathbf{10}\}$. Represent the 3 signals obtained at the output of the filter.

Pay attention: for easy comparison, draw the three signals on the same graph, one under the other.

- for each frequency, C_x will be modified so that the same number of periods (3..4 periods) fit on the scope. It is obviously noy possible to really obtain 3 simultaneous images with different values of C_x.
- C_x, index {1,2,3} and the ground level will be written beside each waveform
- the amplitude can be drawn higher/lower than the real one, for the purpose of fitting the 3 representation on the same graph.

Explain: why are the circuits called integrator/differentiator?

Hint: Think about the analytical expression of a rectangular wave (the analytical expression of a sine wave is found on page 1), then think about the waveform of the output signals in each of the 3 cases.

Preparatory questions

Attention! Bring with you a scientific calculator for the test !
Note: Use of a "smartphone", capable of storing text and images, instead of a calculator is not acceptable during the test.

1. Transform the following voltages into decibels: U_1=2V, U_2=5V, U_3=10V, U_4=200mV, U_5=800mV, U_6=400mV. Reference voltage is U_{ref}=1V. Repeat for dBm (U_{ref}=0.775V)
2. Deduce the expression of the transfer function, its magnitude and argument for the circuits in Figure 2.
3. Determine $U_1/U_2|_{dB}$, if U_1/U_2=20.
4. Determine U_1/U_2, if $U_1/U_2|_{dB}$ =34 dB.
   Hint: lg 2 ≈0,3; lg 3 ≈0,477; lg 5 ≈0,7;

Exercises

1. Calculate the magnitude of the transfer function (frequency response) for circuits in Figure 2, at frequencies $f_t/10$, $f_t/4$, f_t, $2f_t$, $4f_t$, $10f_t$.
2. Calculate the magnitude of the frequency response at frequencies $f_t/10$, $f_t/\sqrt{3}$, f_t, $\sqrt{3}\,f_t$, $10f_t$, where f_t is the cutoff frequency of the circuits in Figure 4. The cutoff frequency is the frequency at which the magnitude of the transfer function falls from its maximum by 3dB.

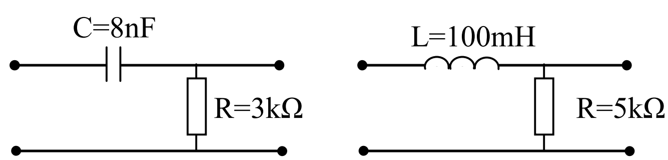


Figure 4

3. Determine and represent the magnitude and the argument of the transfer function ($Z(\omega)=\dfrac{U(\omega)}{I(\omega)}$) of the circuit in Figure 5. Determine its resonance frequency, the frequency at which the magnitude of the frequency response has the maximum value.

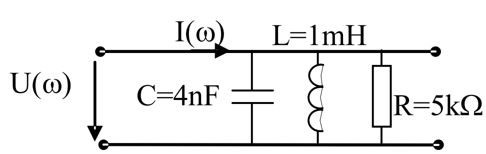


Figure 5

ANNEX 3. AC millivoltmeter

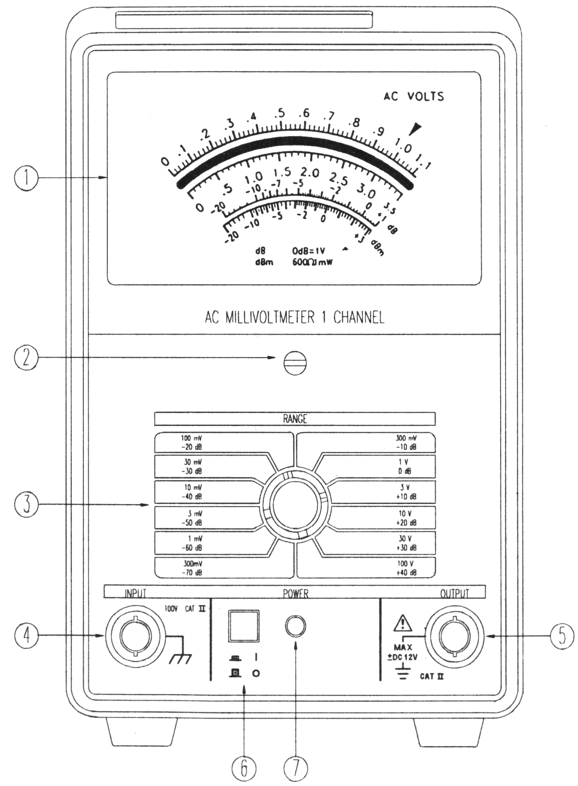


Figure A6. AC millivoltmeter

1. Analog display of scale:
   a) 0 ÷ 1 (extension 1.1), in V.
   b) 0 ÷ 3 (extension 3.5), in V.
   c) -20dB ÷ 0dB (extension +2dB)
   d) -20dBm ÷ 0dBm (extension +3dBm).
2. Zero adjustment.
3. Selector switch – selects the maximum of a scale.
   a) when selecting values of 1mV, 10mV, 100mV, 1V, 10V, 100V, read on scale (a).
   b) when selecting values of 300μV, 3mV, 30mV, 300mV, 3V, 30V, read on scale (b).
   c) To read in dB (U_{ref} = 1V) or dBm (U_{ref}=0,775V), add the value of the indication to the indication of the switch (3).
   $$U\Big|_{dB} = 20 \cdot \lg\left(\frac{U}{U_{\text{Ref}}}\right)$$
4. Input connector (for measuring signal).
5. Output connector.
6. 7 Operating switch and indicator.

Pay attention : The AC millivoltmeter indicates the **true RMS voltage** of the signal and is correct only for **sine** waves. For any other waveform, the device will commit a systematic error.

ANNEX 4 . Digital Multimeter Instek GDM-8246

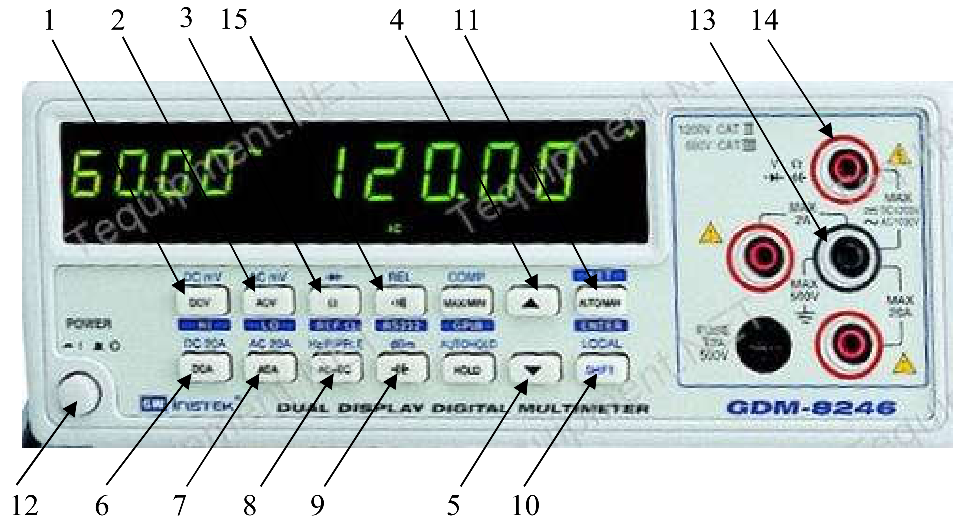


Figure A7. Front panel of digital multimeter

1. key for selecting DC voltage measurement (DC Voltmeter)
2. key for selecting true RMS AC voltage measurement (AC Voltmeter)
3. key for selecting resistance measurement (DC Ohmmeter)
4. key used to increase the value of an internal parameter of the device (or to switch scales in increasing order, on manual mode)
5. key used to decrease the value of an internal parameter of the device
6. key for selecting DC current measurement (DC Ammeter)
7. key for selecting true RMS AC current measurement (AC Ammeter)
8. key for selecting AC+DC measurement
9. key for selecting low frequency capacitance measurement (Capacitance Meter)
10. key for selecting the alternative function, written in blue, for keys 1-9
11. key for switching between automatic and manual modification of scale / entering selection of some internal parameters of the device. Example: selection of the reference resistance in order to indicate the true RMS voltage value in dB or dBm.
12. ON/OFF key
13. negative input terminal (GND)
14. positive input terminal
15. continuity beeper key – when selected, the buzzer sounds when the two terminals are connected; it is used when testing the continuity of some wires, circuits, etc, without watching the display.