Laboratory 6
Impedance measurements
rev 6e

Goal: Impedance measurements using the direct method, with the LCR-meter, or the indirect method, using the DC bridge.

Theory summary

In the sinusoidal regime, the impedance and the admittance are defined as $Z = \frac{\underline{U}}{\underline{I}}$ and $Y = \frac{\underline{I}}{\underline{U}} = \frac{1}{Z}$, where $\underline{U}$ and $\underline{I}$ represent the phasors of the voltage and the electric current intensity, from Fig. 1a.

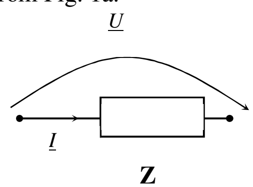

Fig. 1a

Generally, this units are complex values, their *algebraic* form being:

$$Z = R + jX \ , \ Y = G + jB$$

R – the series resistance;

X – the series reactance (with $X > 0$ for inductive impedances, and $X < 0$ for capacitive impedances);

G – parallel conductance;

B – parallel susceptance (with $B > 0$ for capacitive admittances, and $B < 0$ for inductive admittances).

The relationship between the admittance and impedance units are:

$$R = \frac{G}{G^2 + B^2} \qquad X = -\frac{B}{G^2 + B^2}$$

The model of a lossy reactance

Let us consider a lossy reactance, with the quality factor Q, at frequency f. There exist two models for this circuit: series and parallel, shown in Fig. 1b.
X can be the reactance of an inductor, or a capacitor:

$$X_L = \omega L \qquad X_C = -\frac{1}{\omega C}$$

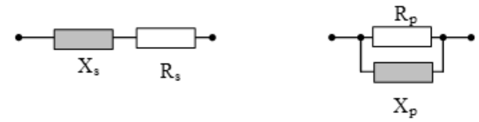

Fig. 1b. The series (left) and the parallel (right) circuit models.

For the two models the quality factors Q_s and Q_p are defined:

$$Q_s = \frac{|X|}{R} = \frac{|X_s|}{R_s} = \frac{\omega L_s}{R_s} = \frac{1}{\omega R_s C_s}$$

$$Q_p = \frac{|B|}{G} = \frac{R_p}{|X_p|} = \frac{R_p}{\omega L_p} = \omega R_p C_p$$

Being defined for the *same* physical component, the two quality factors must be equal:

$$Q_s = Q_p = Q$$

The tangent of the loss angle, D is:

$$D = \frac{1}{Q}$$

The relationships between the elements of the two models, at a given frequency f, are:

$$X_p = X_s\left(1 + \frac{1}{Q^2}\right) = X_s\left(1 + D^2\right)$$

$$R_p = R_s\left(1 + Q^2\right)$$

The equivalence relationship between reactances can be written, in dependence with the nature of the reactance, i.e., capacitive or inductive, as follows:

$$L_p = L_s\left(1 + 1/Q^2\right)$$
$$C_s = C_p\left(1 + 1/Q^2\right) \qquad (1)$$

The principle of quadripolar measurement (4-terminal or 4-wires measurements)

When small valued impedances are measured, or the testing leads are long (measurement at a distance), the impedance of the leads and the contact resistances is negligible no more, being comparable with the impedance Z_x. The measuring principle uses two terminals at each end of the impedance. A pair of terminals is used to inject the current through the unknown impedance Z_x, and the other is used to measure the voltage drop on Z_x. Due to the 4 terminals, the connetion is called *quadripolar*. The two pairs of terminals are to be connected in the proximity of the body of the impedance.

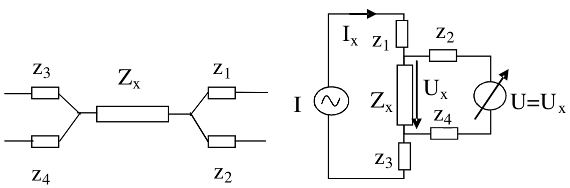

Fig 2a: The quadripolar model.

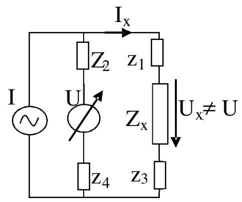

Fig. 2b: The bipolar model.

The quadripolar connection is given in Fig. 2a; two separated pairs of wires are connected:

- from source *I* to Z_x, the (parasitic) impedances of the connection wires are z_1, z_3.
- from the voltmeter to Z_x, the impedances of the wires are z_2, z_4.

One can observe that z_2 and z_4 are in series with the voltmeter which has a very high input impedance such that they are negligible. z_1 and z_3 are connected in series with the current source, with high internal impedance, such that they become negligible too. Therefore, this scheme enables the minimization of the effect of the 4 *small valued* unwanted impedances, by putting them in series with *high valued* impedances already in the circuit.

Fig. 2b shows the bipolar connection. In this case, the measurement instrument contains the source *I* and the voltmeter *U*. Z_x is connected by two wires only, z_1 and z_3. Thus, the paths of the current and the voltage cannot be separated anymore, the measured impedance being the sum of the measured one and those of the wire:

$$Z_m = Z_x + z_1 + z_3$$

causing a systematic error:

$$\varepsilon_{Z_x}{}^s = \frac{z_1 + z_3}{Z_x}$$

For example, when measuring a resistance *R,* by using some cables with resistance r, one obtains a systematic error, when the bipolar (two wires only) configuration is used:

$$\varepsilon_R{}^s = \frac{2r}{R}$$

DC bridges. The Wheatstone bridge.

In Fig. 3 the schematic of a Wheatstone bridge can be seen, where r_g is the internal resistance of the source, and R_V is the internal resistance of the null indicator (or the voltmeter). Considering r_g negligible and R_V very high, the unbalance voltage is given by:

$$U_d = E_g \cdot \left(\frac{R_2}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right)$$

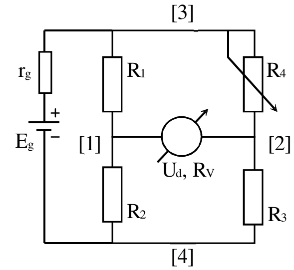

Fig. 3

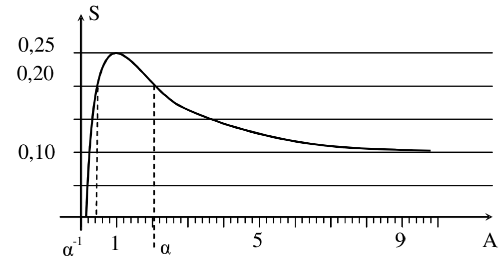

Fig. 4

The bridge is balanced when U_d = U_{12} =0. At the equilibrium, between the resistances of the bridge the relationship (4) is valid:

$$R_1 R_3 = R_2 R_4 \qquad (4)$$

The **ratio of the bridge, *A*,** is defined as the ratio of *any* two *adjacent* resistances which are connected at the same terminal of the *voltmeter*, when the bridge is balanced.

$$A = \frac{R_1}{R_2} = \frac{R_4}{R_3} \quad \text{or} \quad A = \frac{R_2}{R_1} = \frac{R_3}{R_4} \qquad (5)$$

The adjustment factor σ (also called the imbalance of the bridge) is defined as:

$$\sigma = \frac{R_4 - R_{40}}{R_{40}}$$

where R_{40} is the value of the resistance R_4 when the bridge is balanced (resistances R_1, R_2, R_3 are fixed values). For $\sigma \to 0$ the voltage U_{12} can be approximated by using the expression:

$$U_d = S \cdot E_g \cdot \sigma$$

The sensitivity of the bridge, *S*, is:

$$S = \frac{\frac{\Delta U_d}{E_g}}{\frac{\Delta R_4}{R_4}} = \frac{A}{(1+A)^2} \qquad (6)$$

From (6), one can deduce that S(*A)=S(1/A).* This is the reason of choosing ratio *A* to be *either R1/R2* or *R2/R1,* if the 2 resistances are adjacent and connected to the same terminal of the voltmeter.

The dependence of the sensitivity *S* of *A* is depicted in Fig. 4. It can be observed that for A=α, S(α)=S(α^{-1}).

Measurements

1. ***Measuring resistances with the LCR-meter***

The LCR-meter is a device that enables automatic measurement of two parameters of an impedance, at the choice of the user (soft key **MODE**).

- Initialize the LCR-meter using **MENU→ SETUP→ RECALL CALIBRATION→ YES(1) →EXIT.**
- Measure three different valued resistances using the LCR-meter: **SPEED→MEDI**, **DISPLAY→VALUE**, **MODE→ R/Q**, **CIRCUIT→ SERIES**. These settings can be changed by pushing the soft keys situated at the right of the display. The default working frequency is 1kHz (verify on the display; if it is not, push **FREQ** (same as „**-**„), write the desired value and push ENTER). Determine the absolute errors, ΔR, and the relative errors, ε_R, of the value measured by the LCR-meter, *R*, when compared to the nominal value, R_{nom} (the one you find on the measured resistance, explicitly written, or color coded).

$$\Delta R = R - R_{nom},\ \varepsilon_R = \Delta R / R_{nom}$$

2. ***Measuring the resistance of a wire***

Use and compare the bipolar (2T) and the quadripolar (4T) configurations to measure a very small valued resistance (the resistance of a wire, which is below 1 Ohm).

a) connect any available wire to the testing leads of the LCR-meter. Observe that the adapter connected at the LCR-meter uses 4 terminals, the measurement being quadripolar. Each of the two crocodile clips connects 2 terminals of the LCR-meter at one terminal of the unknown impedance, as seen in Fig. 2a. Write down the value for the resistance of the wire.

b) Measure the same wire using the ohmmeter (multimeter, key **Ω**), which only has 2 terminals so the measurements are bipolar. Write down the indicated value, $R_{bipolar}$. Compute the relative error of the bipolar measurement when compared to the quadripolar one. Which is the reason for such a great difference between the values measured at points ***a*** and ***b***?

c) Connect the two alligators of the ohmeter, directly to each other (without the wire between them). Write down the indicated value, $R_{connection\ wires}$. Note this is the systematic absolute error (ΔR) when measuring the wire.

d) Determine the value of the resistance of the measured wire by applying a correction of the systematic error, R_{wire}, as follows: subtract the value of the resistance of the alligator cables from ***c*** from the value measured at point ***b***.

Compute the relative error of this value when compared to the value determined at point ***a***. This value is smaller, greater or equal to the relative error from ***b***? Why? Which is the reason for which at point ***a*** you didn't have to determine the resistance of the connection wires?

3. ***Measuring capacitors and inductors***

a) *measuring capacitors*

Measure two capacitors (of different types and values):

- a stiroflex (polistirene) capacitor between 100 ... 400pF
- a multilayer ceramic capacitor between 1nF... 100nF

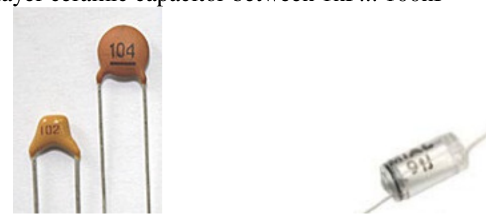

Fig. 5 ceramic capacitors stiroflex

Use **MODE→C/D**, series model (**CIRCUIT→SERIES**) and write down the values C_s and D. Select the parallel circuit (**CIRCUIT→PARALL**) and write down the value C_p. The value of the D factor remains the same. Compute $Q = \frac{1}{D}$

Which capacitor type has the higher Q?

Compare the values C_s and C_p. Justify the results of the comparison.

Determine the parasitic resistance of the capacitors: set **MODE→C/R** and write down the parasitic resistance for the series model (R_s). Based on the relationship between R_s and R_p, and computed Q, determine the parallel resistance R_p. Which of R_s and R_p is greater?

b) *measuring inductances*

Measure the inductance existent at the working station. Set **MODE→L/Q** and measure the series model (L_s and Q), (**CIRCUIT→SERIES**), and the parallel model (L_p and Q), (**CIRCUIT→PARALL**). Compute the quality factor Q_{calc} from the relationship (1) between L_s and L_p. Measure the value of the resistance for the series model R_s. What is the physical cause of R_S ?

Compare the Q factor for inductances and for capacitors.

4. ***Measuring a RC group***

a) Measure a resistor around 10 kΩ on mode C/R, and the smallest capacitor available at the table (< 500pF) on C/D mode (series: C_S , D)

Use them to build a series RC group on the solderless breadboard. Connect the 2 crocodile clips of the LCR-meter directly to the end terminals of the series group (don't use additional wires).

Follow the steps at point 3, to measure the group. The working frequencies are 1kHz and 100KHz. Determine the elements of the series model (C_s, D) and of the

parallel one (C_p, D). Determine $Q = \frac{1}{D}$. Compute the quality factor Q_{calc} of the group using the relationship between the series and the parallel model (1).
Determine the resistance of the group, using C/R mode, series (R_S) and parallel (R_P).
Also compute $X_C = 1/\omega C_S$ at the 2 frequencies.

b) repeat the measurements for a parallel group built on the breadboard using the same R,C values.

c) explain the results:

- what happens to the Q of the group compared of the Q of the capacitor? In which case (S/P) the change is greater, and why?
- we measured a *real* series or parallel group, physically built from 2 components in series or parallel, using the LCR-meter set to the *equivalent* (mathematical) S/P model. In which case the values we measured for C are closer to the value of C measured independently (without R) ?
- at the 2 frequencies, in which case the measured C values differ to a greater extent from the independent C value? Note that greatest errors are not at the same frequencies (large or small) in the two cases (S/P). Explain why, taking into account the value of X_C.

5. *Measuring resistances with the DC bridge*

a) *balancing the bridge*

On the breadboard, build a DC bridge. The nominal values for resistances R_1, R_2, R_3 are [1..5kΩ], 10kΩ and 10kΩ. R_4 is a variable resistor (also called *trimmer* or semi-adjustable potentiometer). Some examples of trimmers, and their equivalent schematic, are given in Fig. 7. The resistances measured between the three terminals 1,2,3 of a trimer follow the rule:

$$R_{12} + R_{23} = R_{13} = ct.$$

where the value R_{13} is the nominal value of the trimmer (a fixed value, independent of the adjustment). The central terminal 2 is called *a cursor* and its position depends on the adjustment screw. Thus, for a 1KΩ trimer, depending on the position of the cursor, one can measure, for example:

$$R_{12} = 100\Omega,\ R_{23}=900\Omega,\ R_{13}= 1K\Omega$$

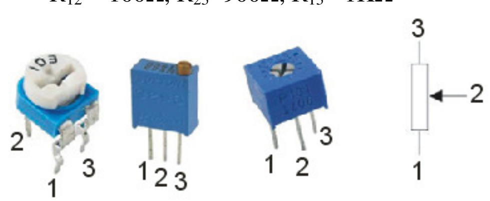

Fig. 7 A trimmer observing the rule $R_{12} + R_{23} = R_{13} = ct.$

Observe that, if either the terminals 1-2 or 2-3 are connected in the circuit, depending on the adjustment, one can measure a resistance between 0 and max value (R_{13}). Do not connect terminals 1 and 3, because trying to adjust the potentiometer is useless, in this case. Observe that the remaining terminal can be connected at the cursor, as in Fig. 3, or it can be let unconnected, with the same result- either way, from an equivalent point of view the trimer must have *two* terminals in the circuit of the bridge.

Fig. 8 gives one option for the circuit from Fig. 3 on the solderless breadboard. The wires from the left part of the schematic are used to connect the DC supply, set at 3V.

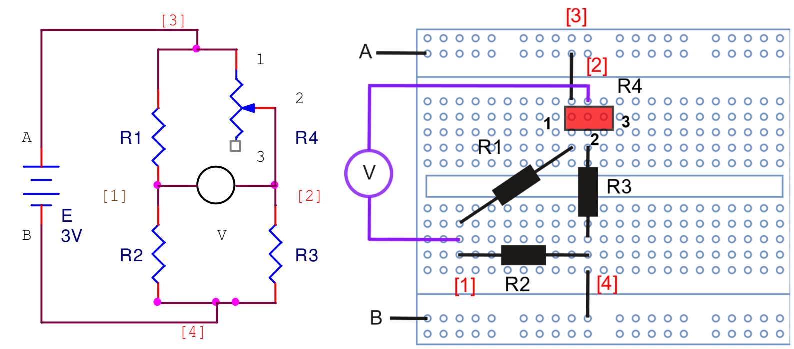

Fig. 8: Example Wheatstone bridge on the protoboard

- Measure the 3 resistances (before connecting them in the circuit). Draw the schematic of the bridge, putting on the scheme the chosen resistances and their *measured* values. Write down the ends of the two diagonals of the bridge (1-2 and 3-4). If you don't write the measured value on each resistance (observing their positioning on the breadboard), you will not be able to identify the two nominal-valued 10k resistances, once you've disconnected them from the LCR-meter!
- Set the DC supply at E=3V (from the switch). Measure with the DC voltmeter, the exact value of the supply voltage, E. *Remark:* for this application, the polarity of the supply voltage is not relevant.
- Connect the supply in a diagonal of the bridge and the voltmeter in the other diagonal. Balance the bridge, adjusting the potentiometer until the indication of the voltmeter is U_d =0 (or the smallest value, in absolute value, that can be practically obtained).

Remark: By adjusting the trimer at the 2 extreme values, you should obtain both positive and negative voltages, and 0V when the bridge is balanced. If, no matter how much you adjust the trimer, the sign of the voltage doesn't change, it means you didn't wired correctly the bridge, and it cannot be balanced.

-Measure the value of the trimmer for which the balance was obtained; do not remove the trimmer, however make sure it is electrically disconnected from the circuit (for instance, temporarily remove R3 and disconnect the power supply). If you don't separate the trimmer from the rest of the circuit, the measurement will be wrong, since you can see that the series group R_1+R_2+R_3 is in parallel to R_4. Use two wires going from the holes in the protoboard corresponding to the trimmer, to the ohmmeter. Note the 2 terminals of the trimmer which were used (2 out of 3: for instance on fig. 8, terminals 1 and 2 were used). Measure the balance value $R_{40\ measured}$

- Compare the value $R_{40\ measured}$ for the potentiometer (when the bridge is balanced) with the value for balance $R_{40\ calc}$ computed with the relation for balanced bridges (4), using the *measured* values for resistances 1,2,3.

Explain the reason for the difference between the 2 values, while the two, $R_{40\ measured}$ and $R_{40\ calc}$ are based on the values of the resistances measured with the same ohmmeter, so the ohmmeter error is not relevant. Which are the sources of this error?

Remark: the voltmeter has a floating input – none of its input plugs is connected to the case; the DC supply has the output floating. If a non-floating supply and a non-floating voltmeter were used (for example, the c.a. millivoltmeter with BNC plug and the c.a. generator with BNC output), the grounds of the two apparatus would be common: the ground would appear between points 1 and 4 (or 2,4) from Fig. 3, putting in short-circuit the resistor R_2, or R_3.

b) *determining the configuration for the maximum sensitivity*
The relation (4) is the same, no matter the diagonals in which the voltmeter and the DC supply are connected. Determine the experimental diagonal that ensures maximum sensitivity (6). For the configuration already wired on the breadboard, reconnect the potentiometer and *slightly* vary it until the voltmeter indicates U_{d1}=20mV. After that, switch the voltmeter and the DC supply (by connecting the voltmeter on diagonal 3-4 and the supply in the diagonal 1-2) and, *without any adjustment of the value of the trimer,* read the indication of the voltmeter U_{d2}, which will be different from U_{d1} – smaller or greater. Which diagonal is more sensitive? Justify!

Compute the ratio of the bridge *A* (relation (5)) and its sensitivity *S* (relation (6)) for the two situations, A_{1-2}, S_{1-2}, A_{3-4}, S_{3-4}, based on the values measured for the 4 resistances. For which of the two cases the computed sensitivity is greater? For what value of A, S is maximum? Compare with the experimental deductions.

Preparatory questions

1. For an inductor L_p=400mH and Q=50 are measured, at the frequency f=kHz. Determine the resistance R_p and the value of the inductor for the series model, L_s.
2. Compute the quality factor for a RC series group, with C_s=10nF and R_s=50Ω, at the frequency 1kHz.
3. Compute the quality factor for a RC parallel group, with C_s=10nF and R_p=1MΩ, at the frequency 1kHz.
4. For an inductor L_s=10mH and Q=10 are measured, at the frequency f=1kHz. Determine the resistance R_s and the value of the inductor for the parallel model, L_p.
5. For a capacitor C_s=200nF and Q=1000 are measured, at the frequency f=10kHz. Determine R_s and the tangent of the loss angle, $D = tg\delta$.
6. A resistor is measured using the bipolar connection (two terminals only). The value of the resistance is R=50Ω. The resistance of the cables is 0,5Ω. Determine the systematic error when measuring the resistor.
7. For the bridge in Fig. 10 compute the resistance R_x when the bridge is balanced and the sensitivity S.

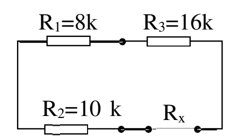

Fig. 9

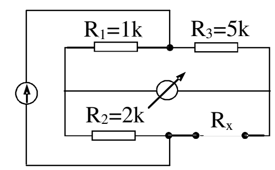

Fig. 10

8. For an inductive impedance, L_p=202mH and L_s=200mH are measured. Determine the quality factor of the impedance.
9. Show that *S* has the same value, no matter the definition for the ratio of the bridge $A = \frac{R_1}{R_2}$ or $A = \frac{R_2}{R_1}$.
10. For a Wheatstone bridge, the imbalance voltage has the values $U_{d1} = 11mV$ for $R_{4.1} = 1{,}011k\Omega$ and $U_{d2} = -11mV$ for $R_{4.2} = 0.989k\Omega$. Determine the value for R_{40} to balance the bridge.
11. Establish the diagonal in which the voltmeter should be connected to maximize the sensitivity of the bridge from Fig. 9. Compute the ratio *A* and the sensitivity, after this choice.